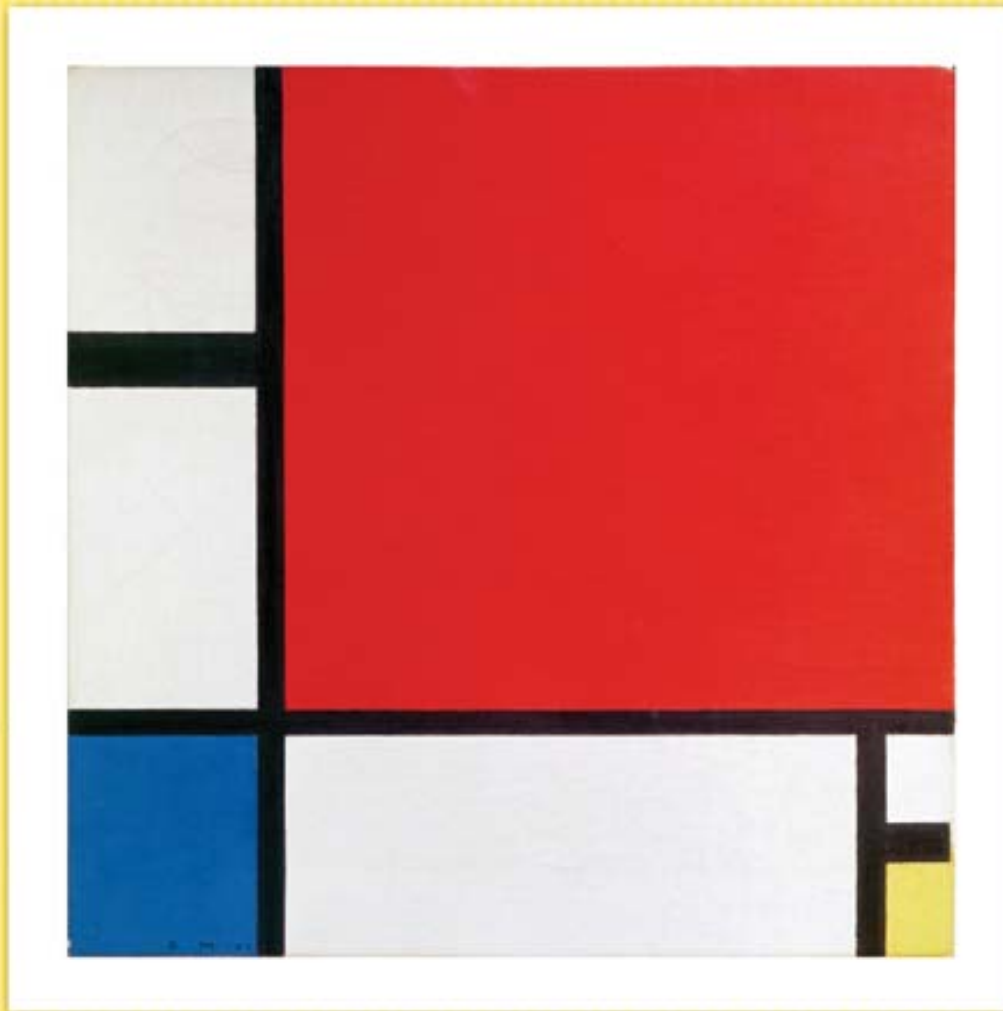


The art of  
**Parallel Programming**  
second edition



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**Bruce P. Lester**

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*Second Edition*  
Bruce P. Lester

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# Brief Table of Contents

## PART ONE

### Shared-Memory Parallel Programming

1 Why Parallel Programming? .....	29
2 Data Parallelism .....	53
3 Multiprocessor Architecture .....	97
4 Process Communication .....	121
5 Data Sharing .....	151
6 Synchronous Parallelism .....	181

## PART TWO

### Distributed-Memory Parallel Programming

7 Multicomputer Architecture .....	225
8 Message-Passing Programs .....	265
9 Data Partitioning.....	309
10 MPI Standard Library.....	361
11 Replicated Workers .....	405
12 Distributed Termination Detection.....	443

## PART THREE

### C\* Compiler and Parallel Computer Simulation System Documentation

Appendix A MPI Function Library .....	481
Appendix B C* Language Guide.....	487
Appendix C User's Manual.....	511
Appendix D Error Messages .....	557

# Index of Parallel Algorithms

## Sorting

Rank Sort	63
Bitonic Merge Sort	140
Bucket Sort	175
Exchange Sort	301
Insertion Sort	283
Merging Sorted Lists	90

## Matrix Algorithms

Matrix Multiplication	68
Saddle Point of a Matrix	175
Cannon's Algorithm	328

## Graph Algorithms

Minimal Spanning Tree	398
Shortest Path	412, 444

## Image Processing

Ray-Tracing	30
Spatial Filtering	375
Edge Detection	377
Region Growing	354
Histogram	161

## Discrete Optimization

Maze Search	437
Traveling Salesman Problem	438, 473
N-Queens Problem	430

## **Numerical Algorithms**

### Solution to Linear Equations

- Back Substitution 130
- Gaussian Elimination 216, 357
- Jacobi Iteration 302, 387
- Asynchronous Iteration 473

### Numerical Integration

- Trapezoid Method 165
- Random Sampling Method 369

### Solution to Laplace's Equation

- Jacobi Relaxation 186, 341
- Red-Black Relaxation 215

### N-Body Problem 313

### Random Number Generation 371

### Prime Number Sieve 144

### Multiplying Polynomials 88

## **Communication Algorithms**

### Broadcast 245

### Aggregation 246

### Multiple Broadcast 294

### Multiple Aggregation 290

### Tournament Technique 194

## **Barrier Algorithms**

### Counting Variable Method 191

### Binary Tree Method 195

### Local Synchronization 201

## **Work Pool Algorithms**

### Termination Detection (shared-memory) 421

### Termination Detection (distributed-memory) 451

### Work Compression 467

# CONTENTS

PREFACE .....	17
---------------	----

## PART ONE

### Shared-Memory Parallel Programming

<b>1 WHY PARALLEL PROGRAMMING?.....</b>	<b>29</b>
1.1 Sequential vs. Parallel .....	29
1.2 Parallel Computer Systems.....	30
1.3 Multiprocessor Architecture .....	32
1.4 Multicomputers.....	36
1.5 Parallel Programming.....	38
1.6 The C* Language .....	42
1.7 Parallel Algorithms.....	45
1.8 The C* Debugger and Performance Monitor .....	49
1.9 Parallelism in the Human Computer .....	50
EXERCISES.....	52
<b>2 DATA PARALLELISM.....</b>	<b>53</b>
2.1 FORALL STATEMENTS.....	54
2.1.1 Process Creation .....	55
2.1.2 Process Granularity .....	57
2.1.3 Optimal Group Size.....	60
2.2 EXAMPLE: PARALLEL SORTING .....	63
2.3 NESTED LOOPS .....	66
2.4 EXAMPLE: MATRIX MULTIPLICATION.....	68
2.5 SHARED AND LOCAL VARIABLES.....	72

2.5.1 Statement Blocks with Declarations.....	74
2.5.2 Scope of <i>forall</i> Indices .....	76
2.6 THE FORK OPERATOR.....	78
2.6.1 Process Termination.....	79
2.6.2 The JOIN Statement .....	80
2.6.3 Parallel List Processing .....	82
2.7 AMDAHL'S LAW .....	83
2.7.1 Effects of Sequential Code on Speedup .....	83
2.7.2 Overcoming Initialization Overhead .....	84
2.8 SUMMARY .....	87
PROGRAMMING PROJECTS.....	88
1. MULTIPLYING POLYNOMIALS .....	88
2. MERGING SORTED LISTS .....	90
EXERCISES.....	91
<b>3 MULTIPROCESSOR ARCHITECTURE .....</b>	<b>97</b>
3.1 BUS-ORIENTED SYSTEMS .....	98
3.2 CACHE MEMORY.....	99
3.2.1 Uniprocessor Caches .....	99
3.2.2 Multiprocessor Caches .....	102
3.3 MULTIPLE MEMORY MODULES .....	104
3.4 PROCESSOR-MEMORY INTERCONNECTION NETWORKS .....	109
3.5 INFLUENCE OF THE ALGORITHM .....	111
3.5.1 Rank Sort.....	112
3.5.2 Matrix Multiplication .....	114
3.6 MEMORY HOT-SPOTS.....	115
3.7 SUMMARY .....	116
EXERCISES.....	118
<b>4 PROCESS COMMUNICATION .....</b>	<b>121</b>
4.1 PROCESS COMMUNICATION STREAMS.....	122
4.2 STREAM VARIABLES .....	124
4.2.1 Reading and Writing Streams.....	124
4.2.2 Producer-Consumer Example.....	127

4.3 PIPELINE PARALLELISM .....	128
4.4 SOLUTION TO LINEAR EQUATIONS .....	130
4.4.1 Back Substitution .....	130
4.4.2 Pipeline Algorithm for Back Substitution .....	133
4.5. STREAMS AND STRUCTURED TYPES.....	137
4.6 BITONIC MERGE SORT.....	140
4.7 SUMMARY .....	143
PROGRAMMING PROJECTS.....	144
1. PRIME NUMBER SIEVE .....	144
2. BITONIC MERGE SORT .....	146
EXERCISES.....	147
<b>5 DATA SHARING.....</b>	<b>151</b>
5.1 ATOMIC OPERATIONS .....	152
5.2 SPINLOCKS .....	157
5.2.1 Implementation of Spinlocks.....	157
5.2.2 Spinlocks and Structured Types .....	159
5.3 CONTENTION FOR SHARED DATA.....	160
5.3.1 Histogram of a Visual Image.....	161
5.3.2 Effect of Contention on Performance.....	163
5.4 NUMERICAL INTEGRATION .....	165
5.5 COMPARING SPINLOCKS AND STREAMS .....	168
5.5.1 Equivalent Power .....	168
5.5.2 Busy-Waiting vs. Blocking .....	171
5.5.3 Semaphores .....	172
5.6 SUMMARY .....	174
PROGRAMMING PROJECTS.....	175
1. SADDLE POINT OF A MATRIX.....	175
2. BUCKET SORT.....	175
EXERCISES.....	176
<b>6 SYNCHRONOUS PARALLELISM .....</b>	<b>181</b>
6.1 SOLVING A DIFFERENTIAL EQUATION .....	182
6.2 PARALLEL JACOBI RELAXATION .....	186

6.2.1 Synchronization by Process Termination .....	186
6.2.2 Barrier Synchronization .....	189
6.3 LINEAR BARRIER IMPLEMENTATION .....	191
6.4 BINARY TREE IMPLEMENTATION OF BARRIERS.....	194
6.4.1 Tournament Technique.....	194
6.4.2 Tree Creation Algorithm .....	195
6.4.3 Performance .....	198
6.5 LOCAL SYNCHRONIZATION.....	199
6.6 BROADCASTING AND AGGREGATION .....	205
6.6.1 Convergence Testing.....	206
6.6.2 Implementing Parallel Aggregation .....	207
6.6.3 Parallel Jacobi with Convergence .....	210
6.6.4 Improving the Performance.....	212
6.7 SUMMARY .....	214
PROGRAMMING PROJECTS.....	215
1. RED-BLACK RELAXATION .....	215
2. GAUSSIAN ELIMINATION .....	216
EXERCISES.....	218

## PART TWO

### Distributed-Memory Parallel Programming

<b>7 MULTICOMPUTER ARCHITECTURE.....</b>	<b>225</b>
7.1 PROCESSOR COMMUNICATIONS .....	226
7.1.1 Communication Link.....	227
7.1.2 Routing and Congestion .....	229
7.1.3 Communication Delay.....	233
7.2 MULTICOMPUTER TOPOLOGY .....	236
7.2.1 Line and Ring Topology.....	236
7.2.2 Mesh Topology .....	237
7.2.3 Hypercube Topology.....	241
7.3 BROADCASTING AND AGGREGATION .....	245

7.4 HYPERCUBE EMBEDDINGS .....	254
7.5 SUMMARY .....	259
EXERCISES.....	261
<b>8 MESSAGE-PASSING PROGRAMS.....</b>	<b>265</b>
8.1 PROCESS COMMUNICATION.....	267
8.1.1 Communication Streams .....	267
8.1.2 Designated Reader.....	270
8.1.3 Comparison with Other Languages.....	272
8.2 LANGUAGE SUPPORT FOR MESSAGE-PASSING .....	274
8.2.1 The MOVE Operation.....	275
8.2.2 Communication with the Main Process.....	276
8.2.3 The @ Primitive for Processor Allocation .....	280
8.3 PIPELINE PROGRAMS ON MULTICOMPUTERS .....	282
8.4 COMMUNICATION DELAY.....	284
8.4.1 Basic Communication Model.....	285
8.4.2 Software Overhead.....	288
8.5 MULTIPLE PORT COMMUNICATIONS .....	290
8.5.1 Multiple Aggregation .....	290
8.5.2 Multiple Broadcast .....	294
8.6 SUMMARY .....	298
PROGRAMMING PROJECTS.....	301
1. SORTING ON A LINE TOPOLOGY .....	301
2. NUMERICAL INTEGRATION ON A MESH .....	302
3. SOLVING LINEAR EQUATIONS ON A HYPERCUBE.....	302
EXERCISES.....	303
<b>9 DATA PARTITIONING.....</b>	<b>309</b>
9.1 COMMUNICATION OVERHEAD .....	311
9.2 N-BODY PROBLEM IN ASTROPHYSICS .....	313
9.2.1 Shared Memory Version .....	313
9.2.2 Partitioning the Data Array .....	316
9.2.3 N-Body Program on Ring Topology.....	318
9.2.4 Performance Analysis.....	320

9.2.5	Overlapping Communication with Computation .....	323
9.2.6	Virtual Vs. Physical Topology .....	324
9.3	MATRIX MULTIPLICATION.....	327
9.3.1	Partitioning the Matrices .....	329
9.3.2	The Algorithm .....	331
9.3.3	Simplified Version .....	333
9.3.4	Complete Program.....	335
9.3.5	Performance Analysis.....	337
9.4	JACOBI RELAXATION .....	341
9.4.1	Multicomputer Algorithm .....	343
9.4.2	Performance Analysis.....	345
9.4.3	Alternate Partitioning Methods .....	351
9.5	SUMMARY .....	352
	PROGRAMMING PROJECTS.....	354
1.	REGION GROWING .....	354
2.	JACOBI RELAXATION ON A MESH .....	356
3.	GAUSSIAN ELIMINATION .....	357
	EXERCISES.....	357
<b>10</b>	<b>MPI STANDARD LIBRARY .....</b>	<b>361</b>
10.1	Process Creation .....	363
10.2	Process Communication .....	365
10.3	Monte Carlo Methods.....	369
10.3.1	Numerical Integration by Random Sampling.....	369
10.3.2	Parallel Random Number Generation .....	371
10.4	Image Processing.....	374
10.5	Group Communication .....	382
10.5.1	Barrier and Broadcast.....	382
10.5.2	Scatter and Gather .....	383
10.5.3	Reduce and All-reduce .....	385
10.6	Solving Linear Equations using Iteration .....	387
10.7	Process Communication Topology.....	391
10.8	Summary .....	396
	PROGRAMMING PROJECTS.....	398

1. MINIMAL SPANNING TREE.....	398
2. MATRIX MULTIPLICATION .....	400
EXERCISES.....	400
<b>11 REPLICATED WORKERS.....</b>	<b>405</b>
11.1 WORK POOLS .....	407
11.2 SHORTEST PATH ALGORITHM.....	409
11.2.1 Sequential Shortest Path Algorithm .....	410
11.2.2 Parallel Shortest Path Algorithm.....	412
11.3 IMPLEMENTATION OF WORK POOLS .....	415
11.4 ELIMINATING CONTENTION .....	417
11.4.1 Load Balancing .....	417
11.4.2 Termination Algorithm.....	421
11.4.3 Performance .....	424
11.5 N-QUEENS PROBLEM .....	430
11.6 SUMMARY .....	435
PROGRAMMING PROJECTS.....	437
1. MAZE SEARCH.....	437
2. TRAVELING SALESMAN PROBLEM.....	438
EXERCISES.....	440
<b>12 DISTRIBUTED TERMINATION DETECTION.....</b>	<b>443</b>
12.1 MULTICOMPUTER SHORTEST PATH PROGRAM .....	444
12.2 WORK POOL IMPLEMENTATION.....	446
12.2.1 Message Acknowledgements .....	448
12.2.2 Termination Detection Algorithm .....	451
12.3 GETWORK AND PUTWORK.....	454
12.4 PERFORMANCE ANALYSIS.....	461
12.4.1 Factors Influencing Performance .....	461
12.4.2 Communication Delay .....	462
12.4.3 Load Balancing .....	466
12.5 WORK COMPRESSION.....	467
12.5.1 New Getwork Implementation .....	467
12.5.2 Asynchronous Algorithms.....	468

12.6 SUMMARY .....	471
PROGRAMMING PROJECTS.....	473
1. ASYNCHRONOUS SOLUTION OF LINEAR EQUATIONS.....	473
2. TRAVELING SALESMAN PROBLEM.....	473
EXERCISES.....	474

## PART THREE

### C\* Compiler and Parallel Computer Simulation System Documentation

<b>APPENDIX A MPI FUNCTION LIBRARY.....</b>	<b>481</b>
BASIC FUNCTIONS.....	481
POINT-TO-POINT COMMUNICATION.....	482
GROUP COMMUNICATION.....	483
CARTESIAN TOPOLOGY .....	485
<b>APPENDIX B THE C* LANGUAGE GUIDE .....</b>	<b>487</b>
B.1 VARIABLE DECLARATIONS.....	488
Arrays.....	488
Pointers.....	490
Structures.....	490
B.2 CONDITIONALS AND LOOPS .....	491
B.3 Input and Output.....	493
Output.....	493
Input .....	496
B.4 FUNCTIONS .....	498
Parameters .....	498
Local Declarations.....	500
B.5 FUNCTION LIBRARY AND PREPROCESSOR.....	502
B.6 ENVIRONMENTS FOR PARALLEL PROCESSES .....	504
B.7 TYPE DEFINITIONS.....	508

B.8 Keywords of the C* Language .....	510
---------------------------------------	-----

**APPENDIX C The C\* Compiler and Parallel Computer  
Simulation System: User's Manual ..... 511**

C.1 GETTING STARTED .....	512
C.2 COMPILATION ERRORS .....	514
C.3 INPUT AND OUTPUT FILES.....	516
C.4 OVERVIEW OF COMMANDS .....	517
C.5 SETTING BREAKPOINTS .....	520
C.6 PROCESSES AND PROCESSORS.....	521
C.7 STEPPING THROUGH A PROCESS .....	525
C.8 WRITING AND TRACING VARIABLES .....	526
C.8.1 Writing Standard Variables .....	526
C.8.2 Writing Structured Variables.....	528
C.8.3 TRACE Command .....	532
C.9 PROGRAM PERFORMANCE STATISTICS .....	534
C.9.1 TIME Command.....	535
C.9.2 UTILILZATION Command.....	538
C.9.3 Sequential Execution Time.....	539
C.9.4 ALARM Command .....	540
C.9.5 VARIATION Command .....	541
C.9.6 DURATION Statement .....	542
C.10 PROGRAM PERFORMANCE PROFILE.....	544
C.11 SIMULATION OF PARALLEL ARCHITECTURES.....	546
C.11.1 Specifying the Architecture .....	546
C.11.2 Communication Delays .....	549
C.12 MPI MODE .....	551
C.13 COMMAND WINDOW PROPERTIES .....	553
C.14 SUMMARY OF COMMANDS .....	554

**APPENDIX D COMPILER ERROR MESSAGES ..... 557**